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The questions may help you, but answering all of them is not compulsory:

*you can simply explain a way to solve an exercise, even if you can't find the solution.*

### Online dating gives women an edge<sup>1</sup>

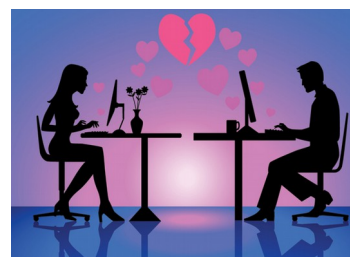
« In 2013, the dating site *Are You Interested* released figures showing that in the United States , a woman sending an online message to a man in her age range had an 18 % likelihood of receiving a response. A man, in the same scenario, had a 4 % likelihood of receiving a response. [...]. Few women who have tried it will deny that online dating makes it easier to be the one to make the first move, that it allows women to be much more selective and forward<sup>2</sup> and that it removes some of the stigma<sup>3</sup> from being the pursuer. »

Time, June 2015

to give somebody an edge<sup>1</sup> : donner un avantage à quelqu'un

forward<sup>2</sup>: effrontée, hardie

stigma<sup>3</sup>: stigmates



<http://i2.kym-cdn.com/entries/icons/original/000/017/323/OnlineDating1.jpg>

#### Part 1 :

According to the CIA, the sex ratio in the USA is 1 male to 1 female among the 25 - 54 years old.

A random experiment consists in selecting at random one person in the 25 - 54 years old.

Calculate the probability that this person gets a response.

#### Part 2 :

Julia and Tom are both Americans (of opposite sex) selected at random. Both have tried to date online by sending 5 messages. Let  $X$  and  $Y$  be the random variables respectively equal to the number of responses Julia and Tom received. We assume that the 5 sendings are independent.

1. Justify that :

- $X$  follows the binomial distribution with parameters 5 and 0.18
- $Y$  follows the binomial distribution with parameters 5 and 0.04

2. Prove that :

- the probability that Julia receives at least one message nears 0.63
- the probability that Tom receives at least one message nears 0.18

3. How many responses can both of them expect on average?

4. Justify the title of this document.

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## THE RENT EXPLOSION IN THE BAY AREA

San Francisco has always been an expensive city to live in, but since 2008 the cost of living has dramatically increased.

From 2011 to 2015, the median price of an apartment has increased by 50% from \$2,595 to \$3,892.5<sup>1</sup>.

1. Given that the rent explosion was constant over the past years, show that the annual growth rate of the median price of an apartment in San Francisco in that period was approximately equal to 10.7%.

From now, let us suppose the rent explosion would follow the same pattern in the coming years.

2. What would the median price of an apartment be in 2016? In 2022?
3. In which year would the 2011 median price have tripled?
4. What is the total amount paid for the rent of an apartment that cost \$2,000 in 2011 and was left in 2015?



Source : <http://priceonomics.com/the-san-francisco-rent-explosion-part-iii/>

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1. Monthly rent.

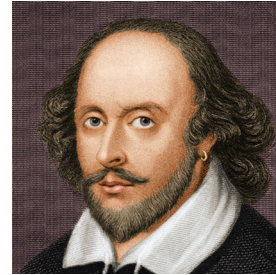
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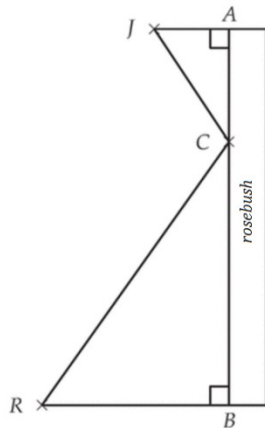
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### Romeo and Juliet

Romeo and Juliet have fallen in love but their parents are enemies and they don't want them to meet. Juliet is really sad waiting on a seat in Capulet's garden, near a wall where roses grow. Romeo sees his beloved one and would like to console her as quickly as possible after picking a rose on the way.



William  
Shakespeare



$J$  represents Juliet and  $R$  Romeo.  $[AB]$  is the wall where the roses grow.  $C$  moves on  $[AB]$ , representing where Romeo will pick the rose before meeting Juliet.

$$AB = 6$$

$$JA = 2$$

$$RB = 5$$

$(JA)$  and  $(RB)$  are perpendicular to  $(AB)$

$C$  is a free point on  $[AB]$

For the route being as short as possible where must  $C$  be?



Claire Danes as Juliet and Leonardo DiCaprio as Romeo

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## Ice cream

In the eighties, a famous brand of ice cream decided to modernize the shape of their products.

Same thickness (1.5 cm),  
but a more rounded look:

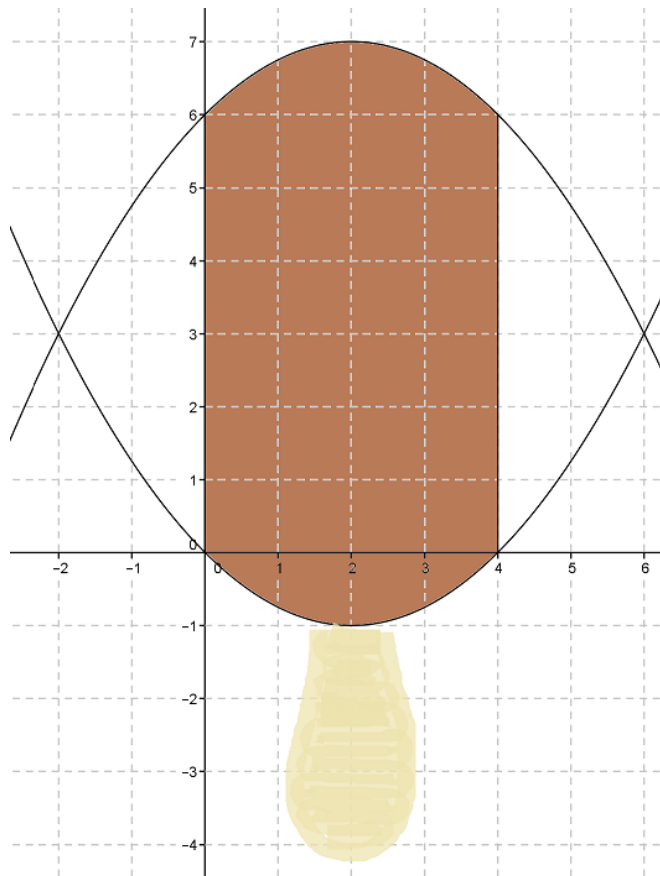
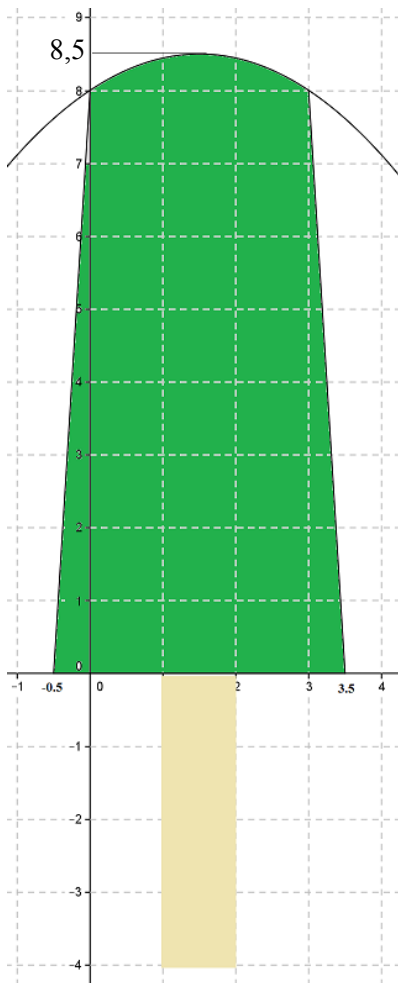
Before



After



The two mathematical models are given below:



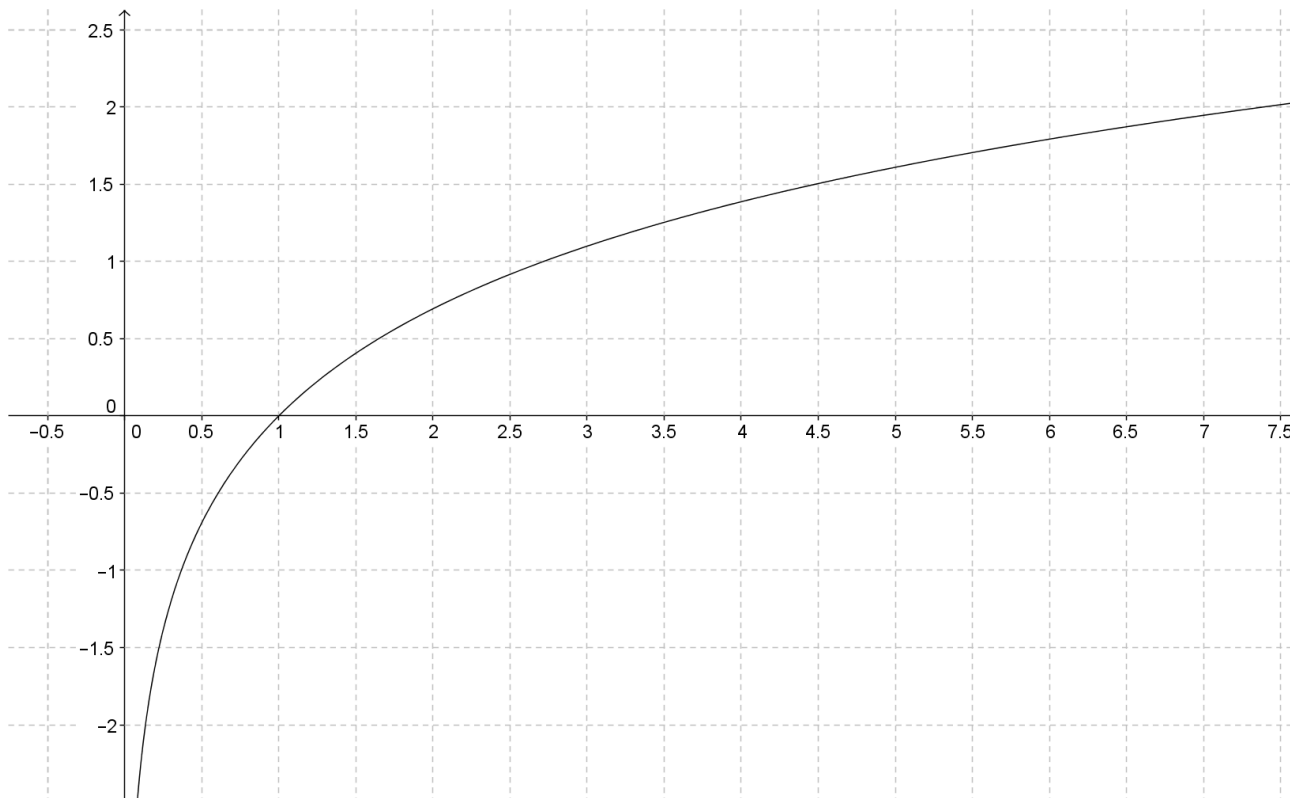
- 1) Match the equations of the parabolas  $y = -\frac{2}{9}\left(x - \frac{3}{2}\right)^2 + \frac{17}{2}$  and  $y = -\frac{1}{4}(x - 2)^2 + 7$  with one of the curves below.
- 2) One unit is one centimetre on both axes.  
Compare the volume of the two models?

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## The natural logarithm function

Let (C) be the curve  $y = \ln(x)$ , let  $a$  be a real number greater than zero and let A be the point of (C) with abscissa  $a$ .

- 1) (i) Write down the equation of (T) the tangent line to curve (C), at point A.  
(ii) What is its y-intercept?  
(iii) Deduce a way to construct geometrically (C)'s tangent line at any point.
- 2) Show that the tangent line (T) goes through the origin point if and only if  $a = e$ .
- 3) (i) in order to compare  $\ln(x)$  and  $x - 1$  for any real number  $x$  greater than 0, study the variations of the function  $g$ :  $g(x) = \ln(x) - x + 1$ ,  $x > 0$ .  
(ii) What can you then deduce about the curve (C)?



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## Salaries

Employees of a real estate agency<sup>1</sup> must choose between two types of payment for their monthly salary :

- **option A:** the salary consists of a fixed amount of \$1,900 plus \$400 per sale.
- **option B:** the salary consists of a fixed amount of \$1,900 plus an increase of 15% for each new sale.

We denote  $A_n$  and  $B_n$  respectively the salary for  $n$  sales.



1. Justify that  $A_1 = 2,300$  and  $B_1 = 2,185$ .
2. Express  $A_n$  in terms of  $n$ .
3. Explain why  $(B_n)$  is a geometric sequence.
4. a) Which option gives the most attractive salary for an employee who makes 9 sales in a month?  
b) Work out the most interesting option, depending on the number of sales made in a month.

A real estate agency<sup>1</sup>: *une agence immobilière*

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## Marathon Man



Mark has decided to train for a marathon which he has to run on May 1<sup>st</sup>, 2015. He has begun to train on September 4<sup>th</sup>, 2014. There are 34 weeks of training. He has decided to run twenty kilometres on the first week and to add four kilometres each week.

- 1) a) Calculate the distance run on the second week.  
b) Calculate the distance run on the tenth week.
- 2) Calculate the distance Mark is going to run the week before the competition.
- 3) The week number, the distance run each week and the total distance run from the beginning of the training are shown in the spreadsheet below. Twenty has been input into cells B2 and C2.

	A	B	C
1	Week	Weekly distance	Added distance
2	1	20	20
3	2	24	44

- a) Give the formulas to input in cells B3 and C3 of the spreadsheet.
- b) Calculate the total distance run by Mark at the end of his training knowing that :  
for all positive integers  $n$ ,  $1 + 2 + \dots + n = \frac{n(n+1)}{2}$ .



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## Useful imaginary numbers

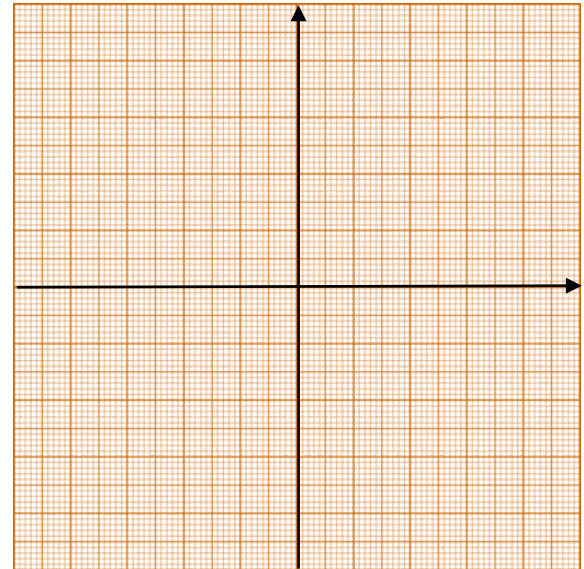
For any complex number  $z$ , we consider the points M, N, P and Q with respective affixes  $z$ ,  $iz$ ,  $i^2z$  and  $i^3z$  :

$$M(z) \quad N(iz) \quad P(i^2z) \quad Q(i^3z)$$

- a) Choose a complex number  $z$ , and plot the corresponding points on the complex plane.

Choose another value for the complex number  $z$ , and do the same again (maybe with another colour).

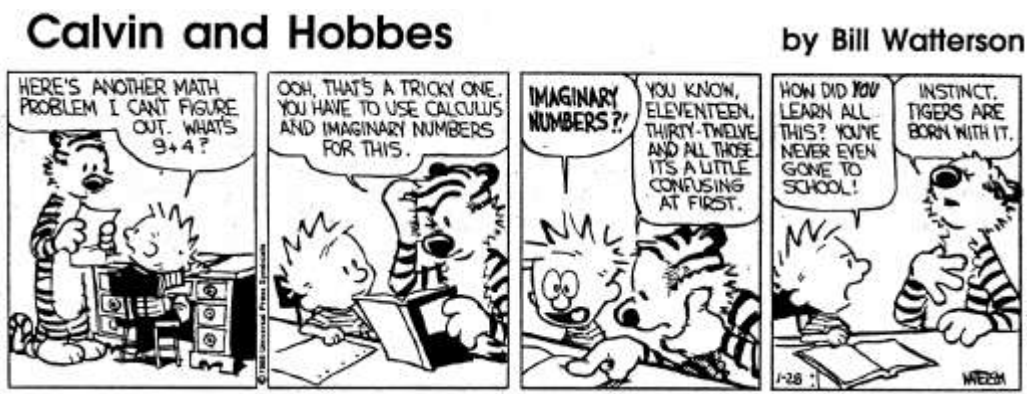
- b) Make conjectures : can you guess something about MNPQ? And about its diagonals? Explain why.
- c) Try to prove your conjectures.



*If you have time...*

Now what could be said about the triangle RST when for any complex number  $z$ , we consider the points R, S and T with respective affixes  $z$ ,  $jz$ ,  $j^2z$  where  $j = e^{\frac{2i\pi}{3}}$  ?

$$R(z) \quad S(jz) \quad T(j^2z)$$



From The Mercury, March 11, 1998, p.31

Complex numbers are useful in many situations... Can you give examples?



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## Dingo vs wallaby



<http://en.wikipedia.org/wiki/Dingo>



<http://hd1wallpapers.com/jumping-wallaby-australia-full-hd-wallpaper/>

In Australia, there are nearly 34 million « kangaroos » issued from different species. The largest species in the family are called « kangaroos ». They have almost no natural predators. The smallest species are generally called « wallabies ». Their main predators are dingos, wild-dogs and foxes.



<http://www.dailytelegraph.com.au/news/nsw/surf-no-safe-haven-for-wallaby-as-dingo-strikes-on-beach-near-yamba/story-e6freuzi-1225968584224>

A dingo is chasing a wallaby. To run away, the wallaby is able to jump many times. But it's quickly getting tired. We assume that its first jump is 3 metres long, and for each new jump, the tiredness effect causes a 2 % loss of length compared to the previous one.

**Questions :** (If needed, round the results to two decimal places).

- 1) Explain why this situation can be modelled by a sequence.  
Let  $(u_n)$  be this sequence; give its characteristics.
- 2) What is the length of the second jump? Of the tenth jump?
- 3) What is the distance the wallaby covers after 15 jumps?
- 4) We assume that the dingo can't chase the wallaby over more than 100 metres, because, after that, it is exhausted... How many times will the wallaby have to jump in order to escape the dingo?